EE 508
Lecture 17

Basic Biquadratic Active Filters

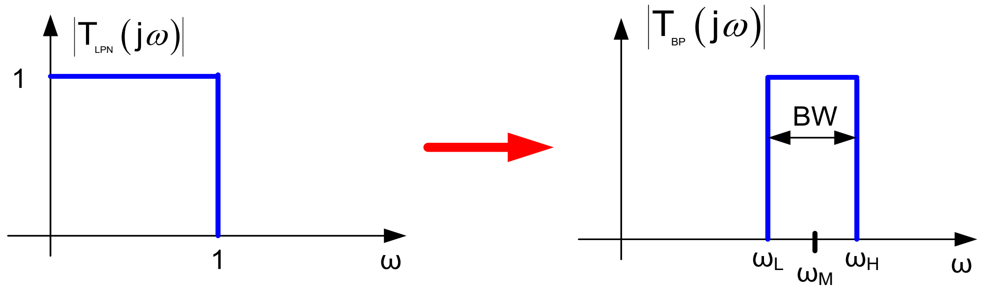
Second-order Bandpass
Second-order Lowpass
Effects of Op Amp on Filter Performance

Review from Last Time

Comparison of Transforms

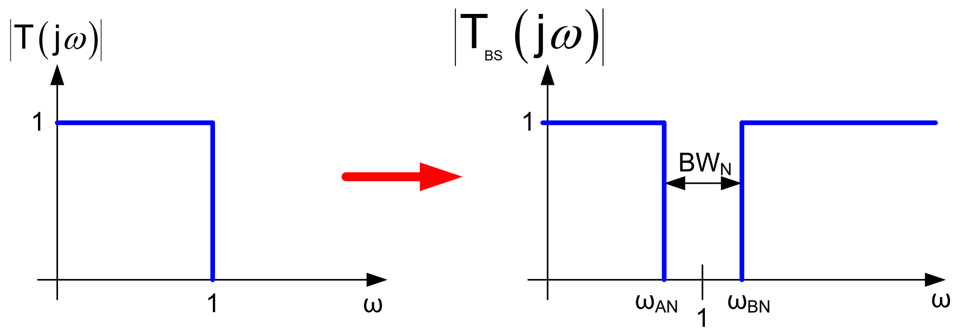
LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$



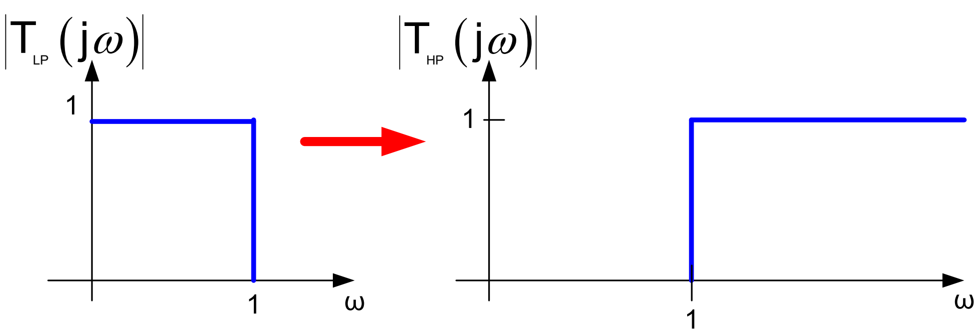
LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$



LP to HP

$$s \rightarrow \frac{1}{s}$$



Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Filter Design/Synthesis Considerations

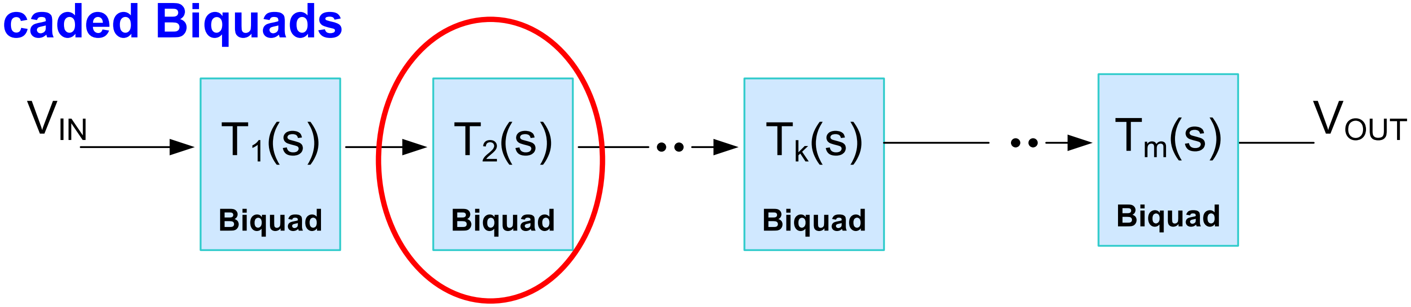
There are many different filter architectures that can realize a given transfer function

Considerable effort has been focused over the years on “inventing” these architectures and on determining which is best suited for a given application

Filter Design/Synthesis Considerations

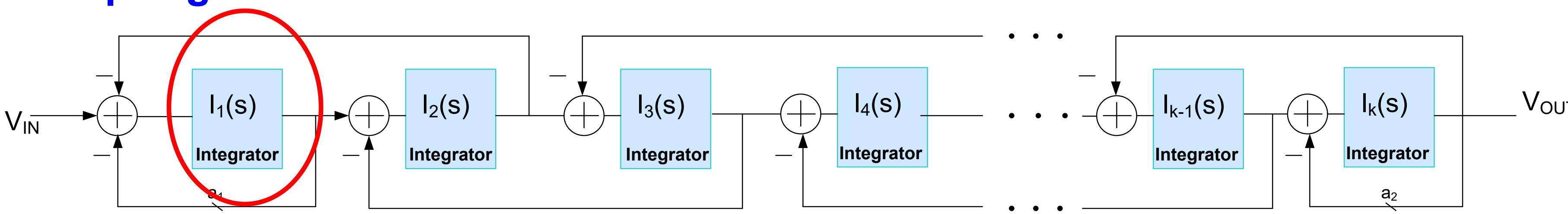
Most even-ordered designs today use one of the following three basic architectures

Cascaded Biquads

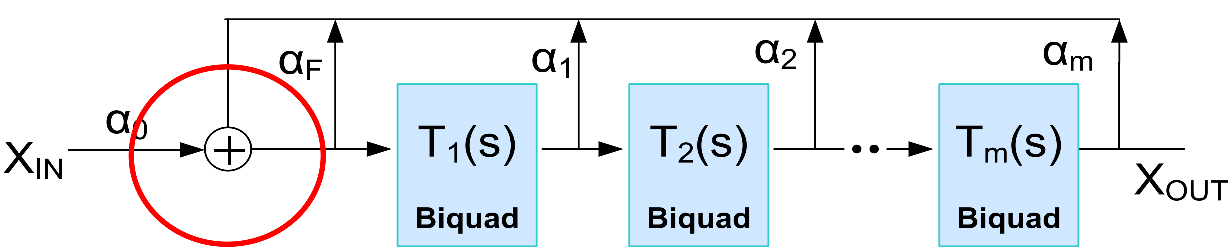


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

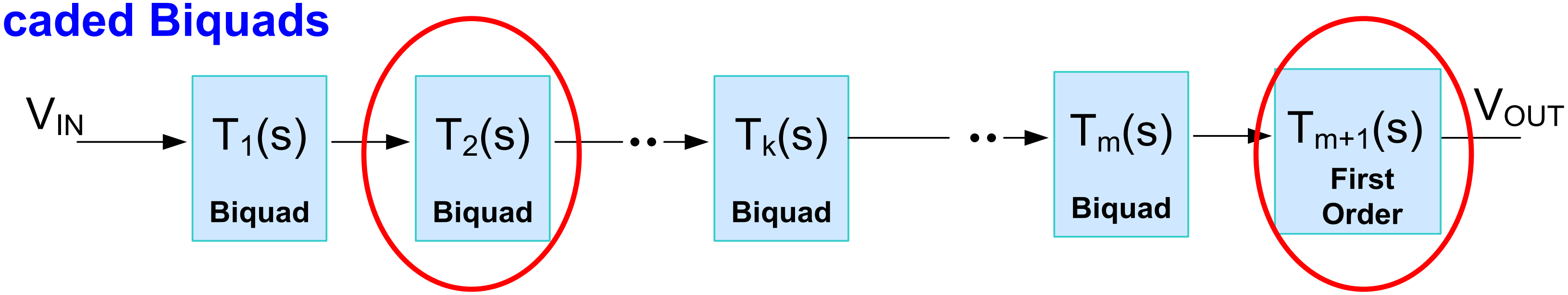


What's unique in all of these approaches?

Filter Design/Synthesis Considerations

Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads



$$T(s) = T_1 T_2 \bullet\bullet T_m$$

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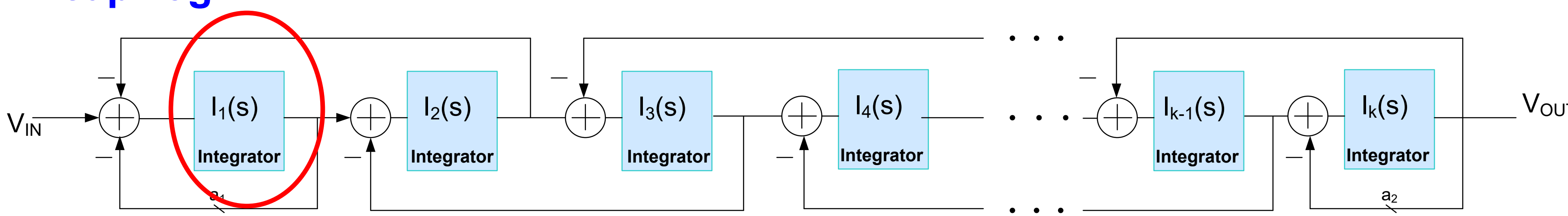
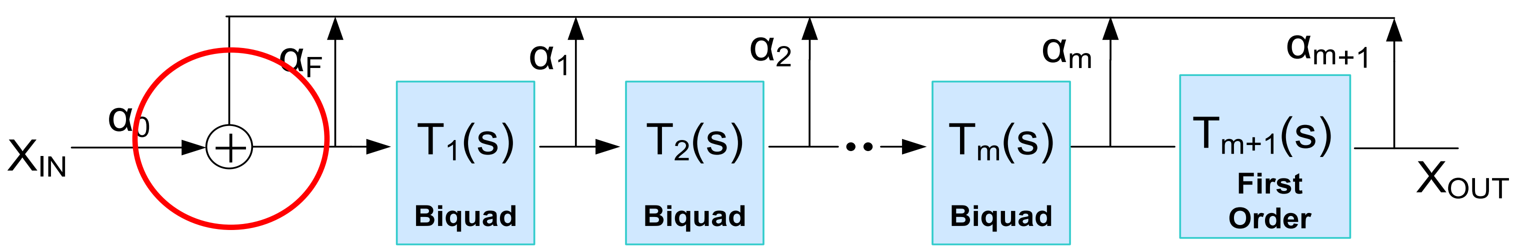


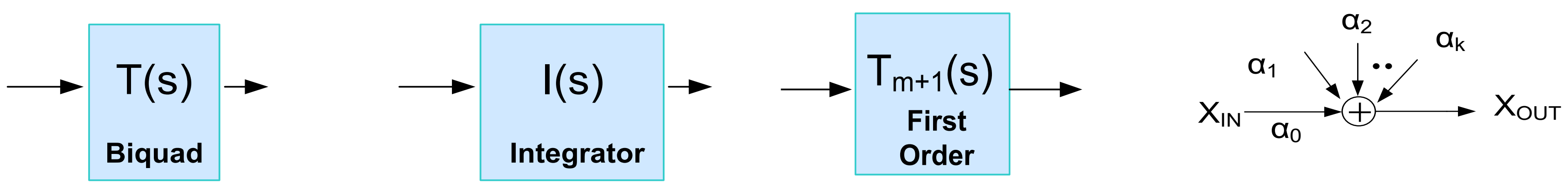
Multiple-loop Feedback (less popular)



What's unique in all of these approaches?

Filter Design/Synthesis Considerations

What's unique in all of these approaches?

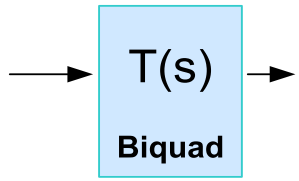


$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad I(s) = \frac{I_0}{s} \qquad T(s) = \frac{a_1 s + a_0}{s + b_0} \qquad X_{OUT} = \sum_{i=0}^{k} \alpha_i$$

- Most effort on synthesis can focus on synthesizing these four blocks
  (the summing function is often incorporated into the Biquad or Integrator)
  (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators
  (thus, much filter design work simply focuses on the design of integrators)

Biquads

How many biquad filter functions are there?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_1 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_1 s}{s^2 + b_1 s + b_0} \qquad a_1 \neq 0$$

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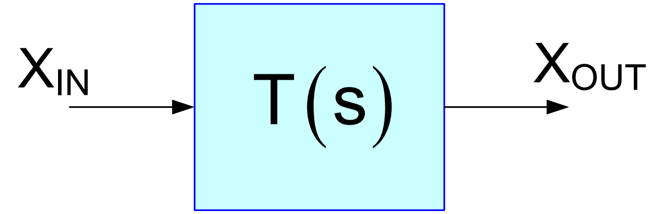
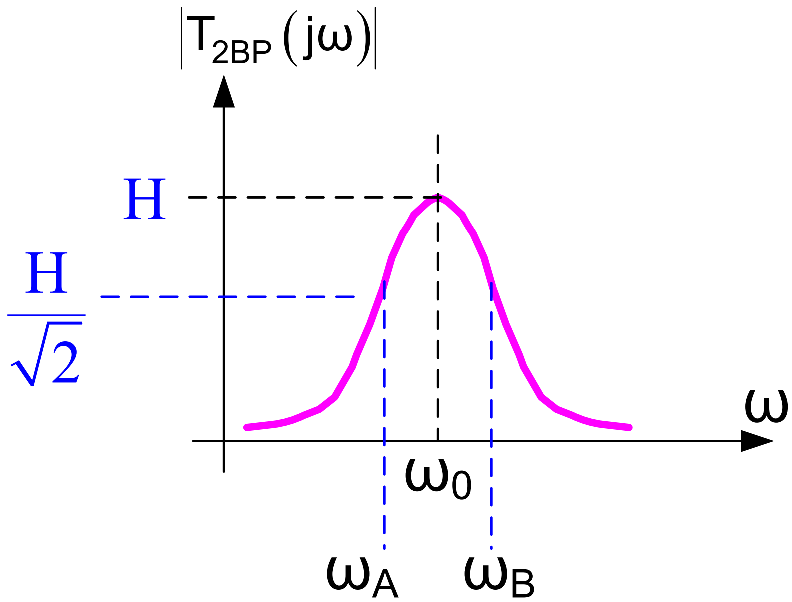
$$T(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_1 s}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0,\ a_1 \neq 0$$

Filter Design/Synthesis Considerations

Review: Second-order bandpass transfer function





$$|T_{2BP}(s)| = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

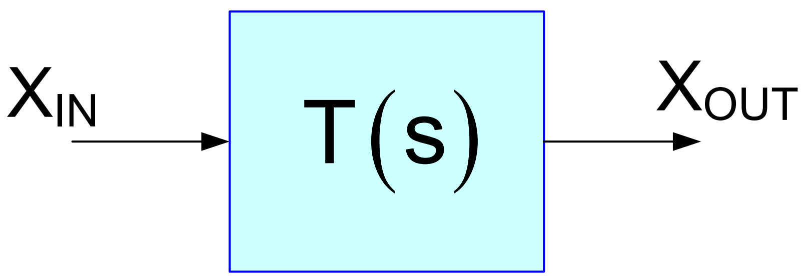
$$\omega_{PEAK} = \omega_0$$

$$\omega_0 = \sqrt{\omega_A \omega_B}$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



$$|T(s)| = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

(unless stated differently assume BW is the 3dB bandwidth)

$$\omega_{PEAK} = \omega_0$$

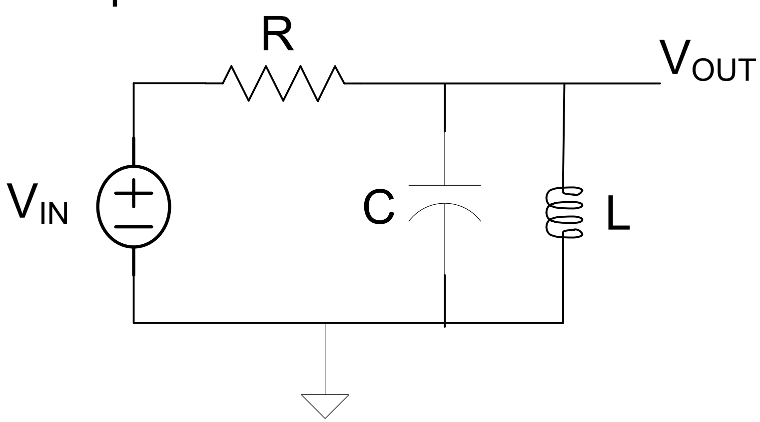
$$\omega_0 = \sqrt{\omega_A \omega_B}$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC} \frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

Second-order Bandpass Filter

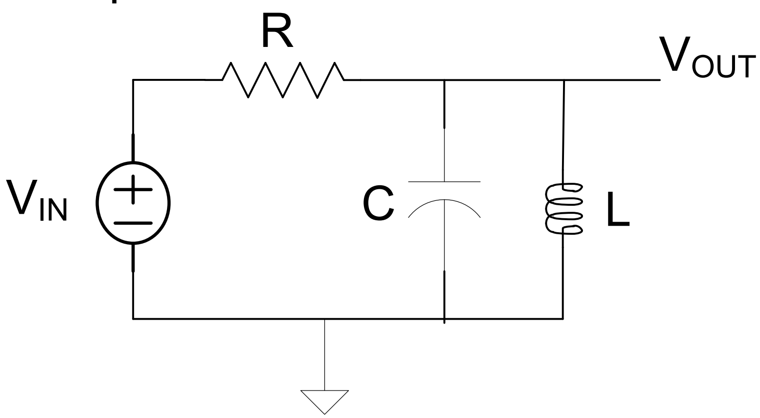
3 degrees of freedom

2 degrees of freedom (RC, LC) for determining dimensionless transfer function

(impedance values scale)

$\omega_0 = ?$ $\quad$ Q = ? $\quad$ BW = ?

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2+s\left(\frac{1}{RC}\right)+\frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

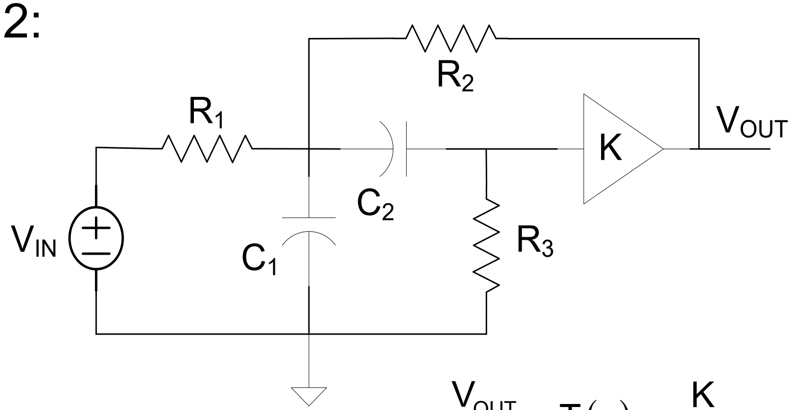
Can realize an arbitrary stable 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

If trimming is necessary, prefer to trim with a single resistor

Can’t trim this filter with single resistor

Example 2:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{R_1C_1}\frac{s}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}+\frac{1\text{-}K}{R_2C_1}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

Second-order Bandpass Filter

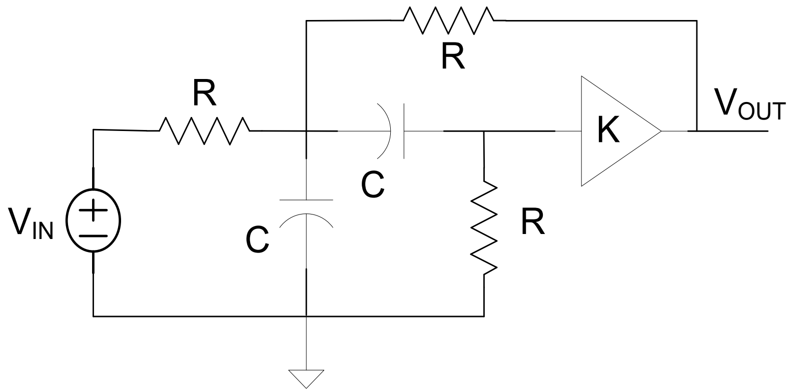
6 degrees of freedom (effectively 5 because dimensionless)

Denote as a +KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Lots of flexibility (6 DOF but complicated expressions for ω_0 and Q)

Example 2 (special case of previous ckt) :



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\,\frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

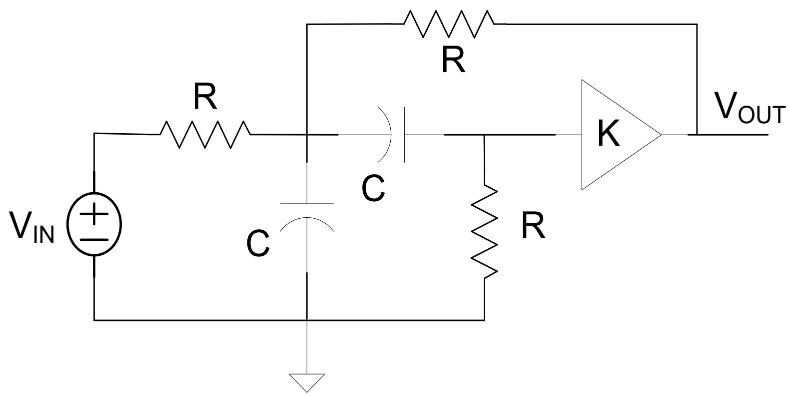
3 degrees of freedom (effectively 2 because dimensionless)

$\omega_0 = ?$

Q = ?

BW = ?

Example 2 (special case of previous circuit) :



Equal R, Equal C Realization

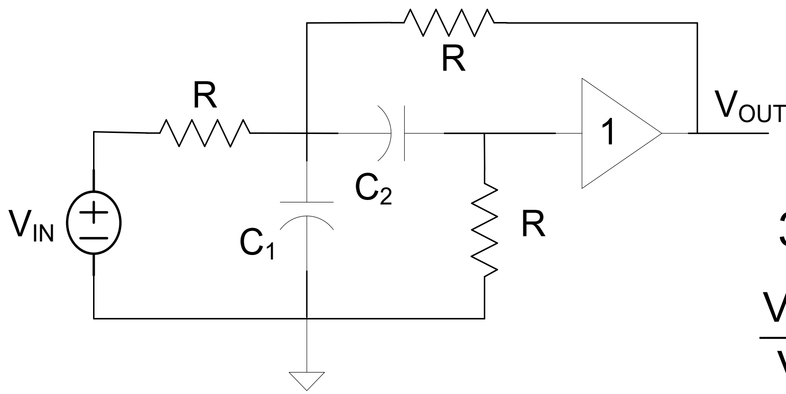
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K} \qquad BW = \frac{4-K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Independent control of ω_0 and Q but requires tuning more than one component
- Can actually move poles in RHP by making K >4

Example 2 (another special case of previous circuit) :



Unity Gain, Equal R

3 degrees of freedom (effectively 2 since dimensionless)

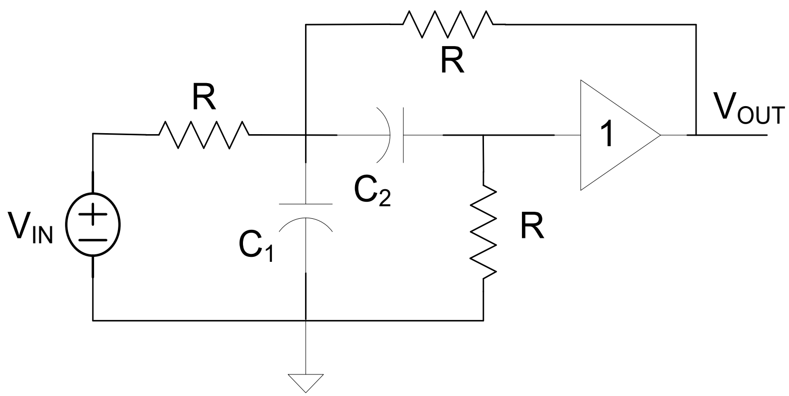
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\frac{s}{s^2+s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1}+\frac{1}{C_2}\right)\right)+\frac{2}{R^2C_1C_2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2 (another special case of previous circuit) :



Unity Gain, Equal R

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

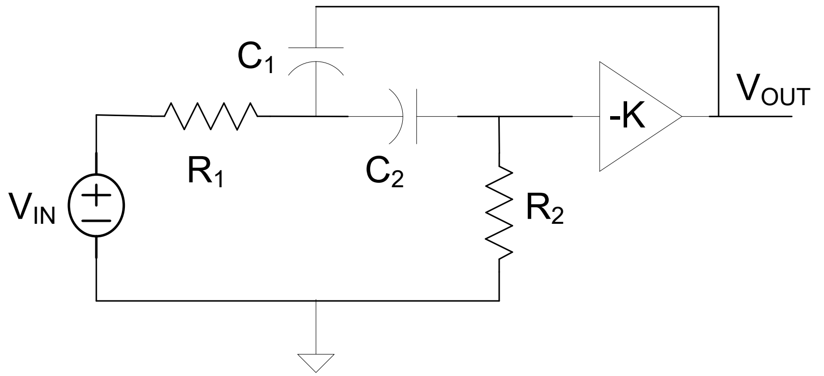
$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1 C_2}}$$

$$Q = \sqrt{2}\sqrt{\frac{C_2}{C_1}} + \frac{1}{\sqrt{2}}\sqrt{\frac{C_1}{C_2}}$$

$$BW = \left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)$$

Can't trim this filter with resistor

Example 3:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)R_1C_1}\frac{s}{s^2+s\left(\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)R_1R_2C_1C_2}}$$

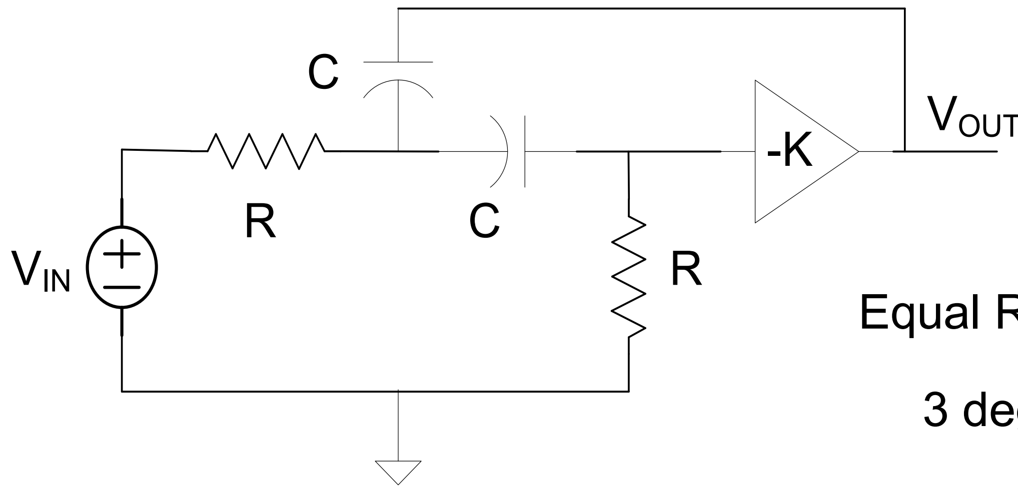
Second-order Bandpass Filter

5 degrees of freedom (4 effective since dimensionless)

Denote as a -KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 3 (special case of previous circuit):



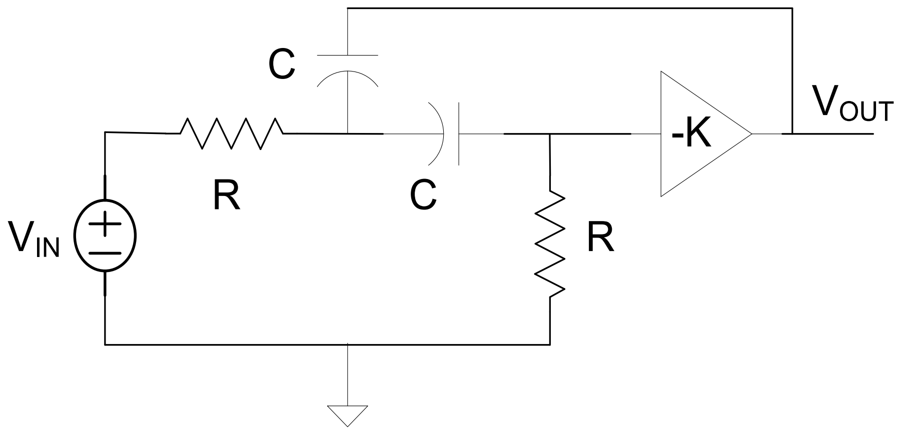
Equal R, Equal C Realization

3 degrees of freedom

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC} \frac{s}{s^2 + s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right) + \frac{1}{(1+K)(RC)^2}}$$

$\omega_0 = ?$ Q = ? BW = ?

Example 3 (special case of previous circuit):



Equal R, Equal C Realization

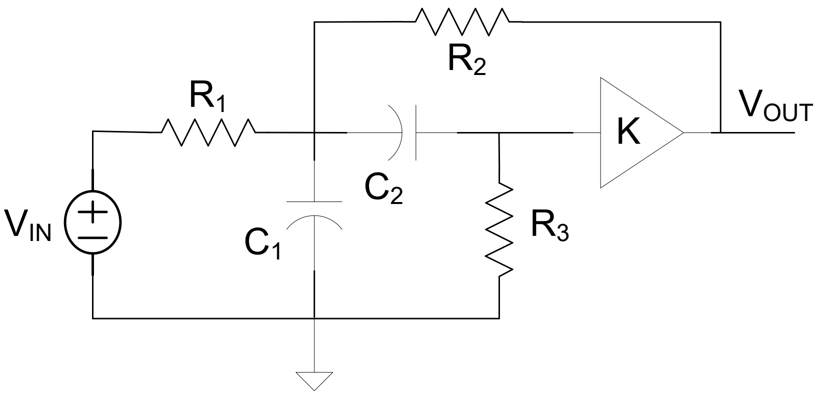
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

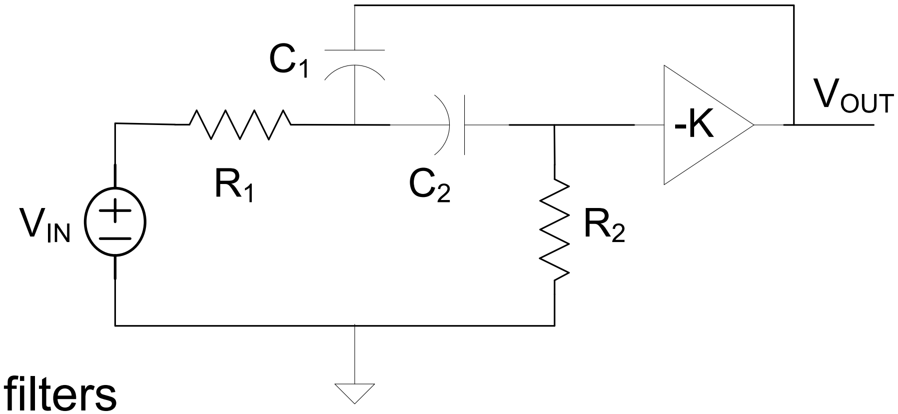
$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

Observation:





These are often termed Sallen and Key filters

Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers,
RC network, and finite gain amplifiers

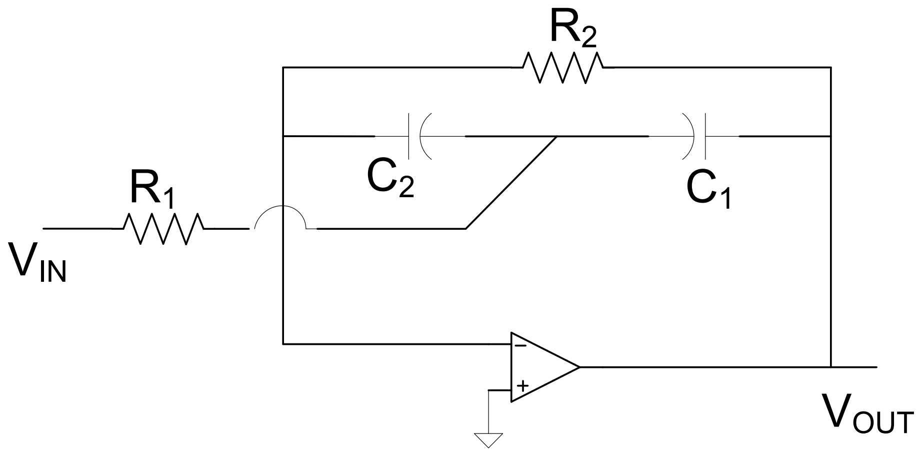
These filters were really ahead of their time and appeared long before practical implementations were available

IRE TRANSACTIONS—CIRCUIT THEORY *March* 1955

A Practical Method of Designing RC Active Filters*

R. P. SALLEN† AND E. L. KEY†

Example 4:



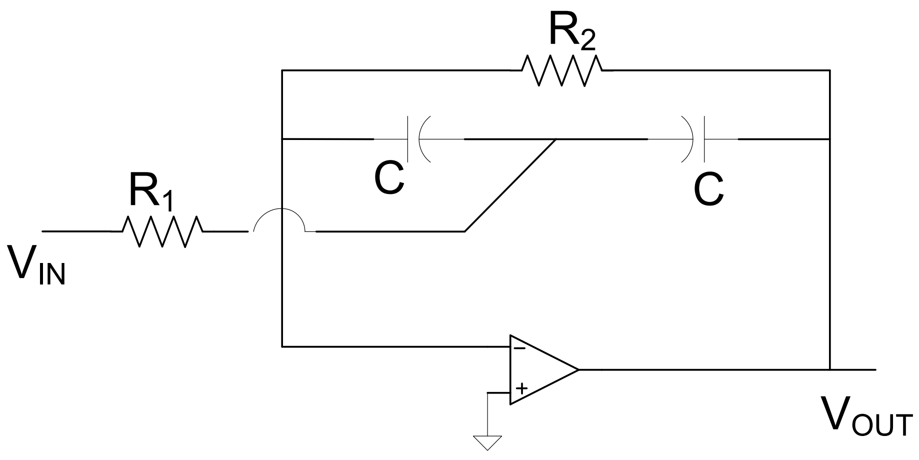
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C_1}\frac{s}{s^2 + s\left(\frac{1}{R_2}\left[\frac{1}{C_1}+\frac{1}{C_2}\right]\right) + \frac{1}{R_1R_2C_1C_2}}$$

Second-order Bandpass Filter

4 degrees of freedom (3 effective since dimensionless)

Denote as a bridged T feedback structure

Example 4 (special case of previous circuit):



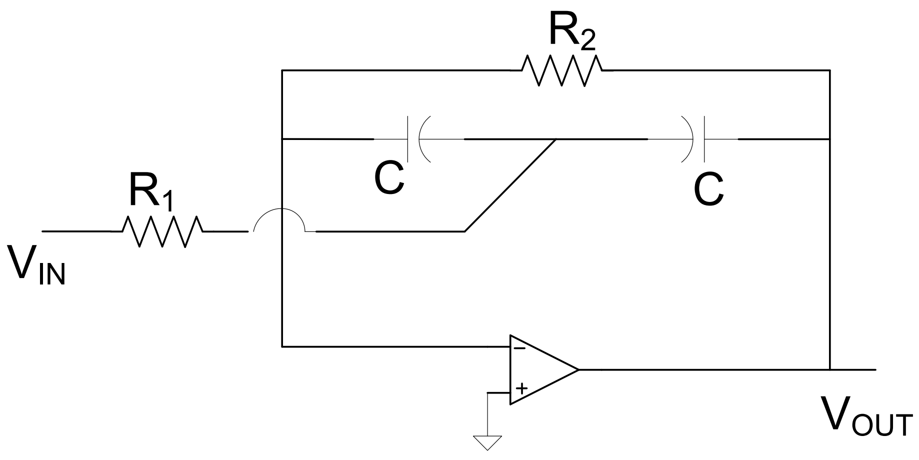
Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C}\,\frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

3 degrees of freedom (2 effective since dimensionless)

$\omega_0 = ?$ Q = ? BW = ?

Example 4 (special case of previous circuit):



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C}\frac{s}{s^2+s\left(\frac{2}{CR_2}\right)+\frac{1}{R_1R_2C^2}}$$

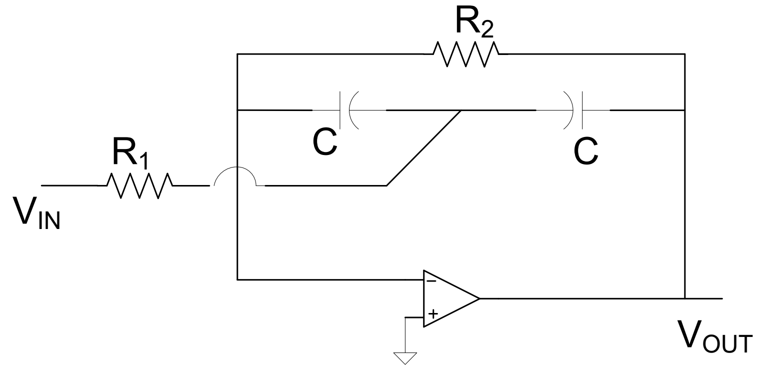
$$\omega_0 = \frac{1}{C\sqrt{R_1R_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \qquad BW = \frac{2}{R_2C}$$

Simple circuit structure

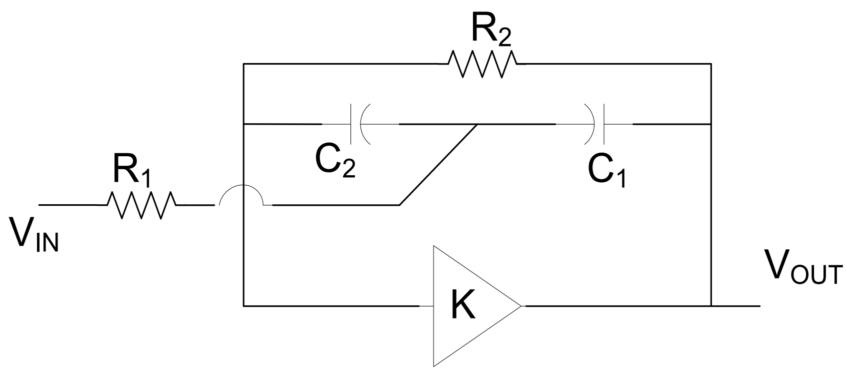
More tedious design/calibration process for ω_0 and Q (iterative if C is fixed)

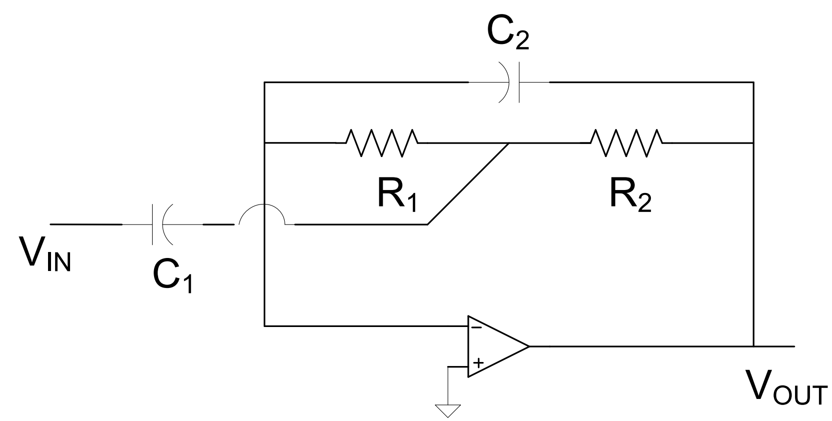
Resistor ratio is $4Q^2$

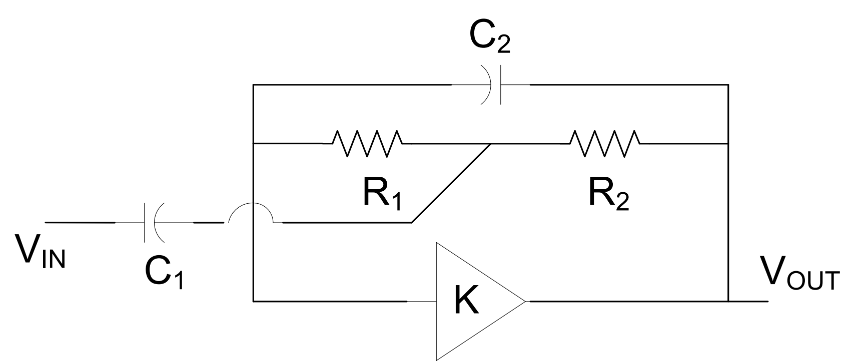
Example 4 (special case of previous circuit):



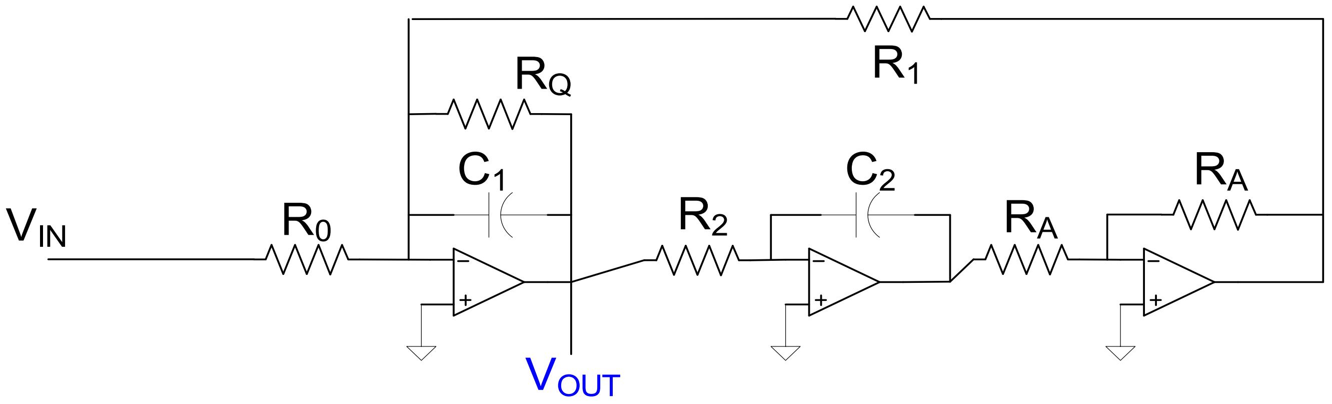
Some variants of the bridged-T feedback structure







Example 5:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0C_2}\frac{s}{s^2+s\left(\frac{1}{R_QC_2}\right)+\frac{1}{R_1R_2C_1C_2}}$$

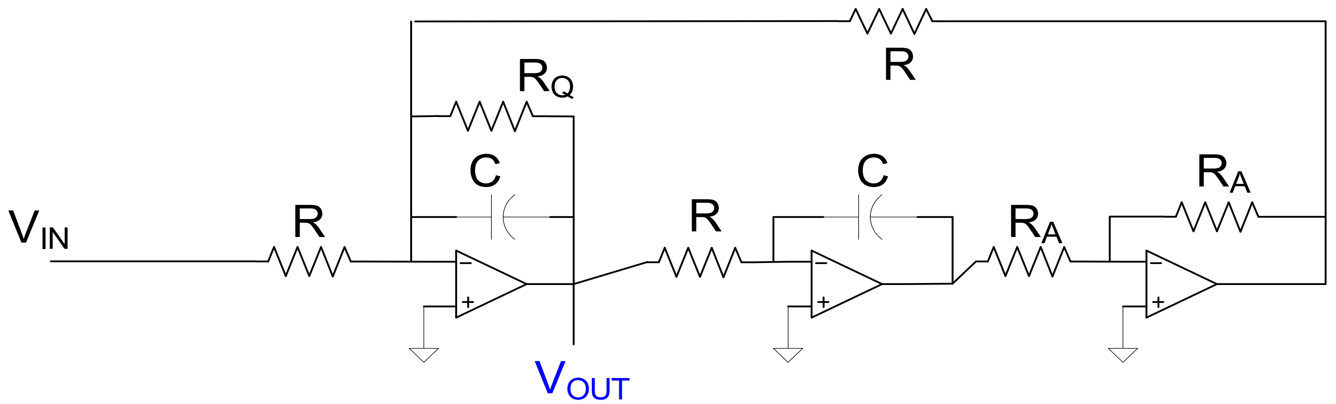
Second-order Bandpass Filter

8 degrees of freedom (effectively 7 since dimensionless)

Denote as a two-integrator-loop structure

Often termed the Tow-Thomas Biquad

Example 5 (special case of previous filter):



Equal R Equal C (except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

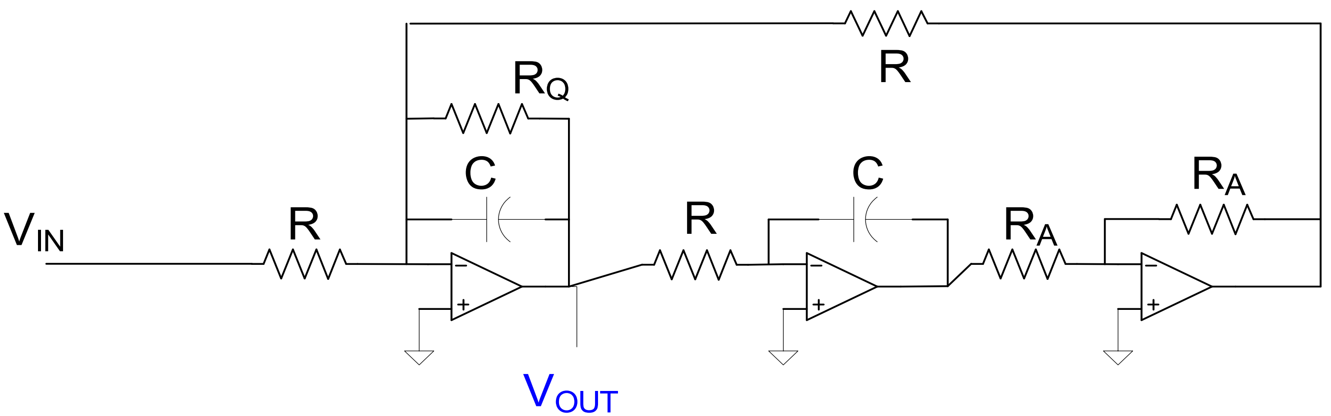
3 degrees of freedom (effectively 2 since dimensionless)

$\omega_0 = ?$

Q = ?

BW = ?

Example 5 (special case of previous filter):



Equal R Equal C (except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

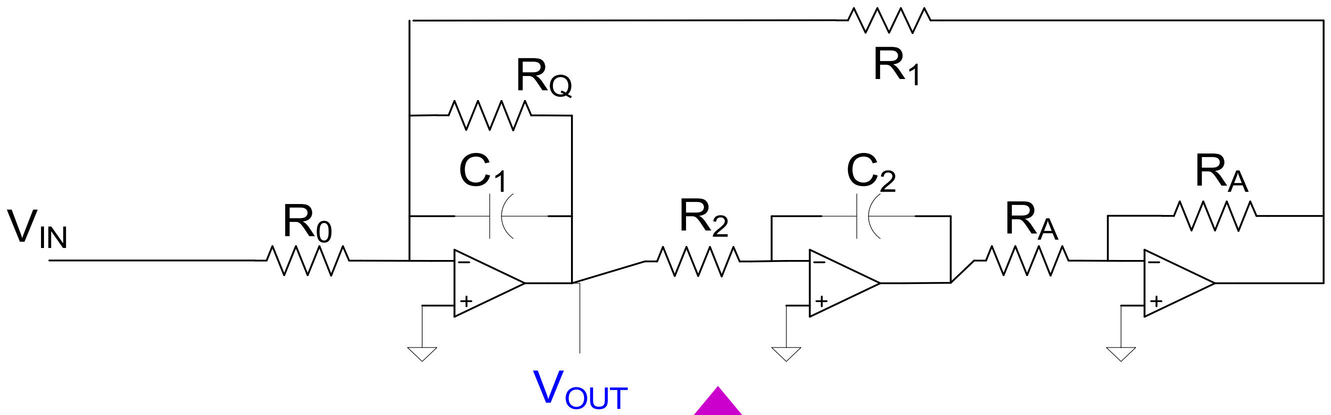
3 degrees of freedom (effectively 2 since dimensionless)

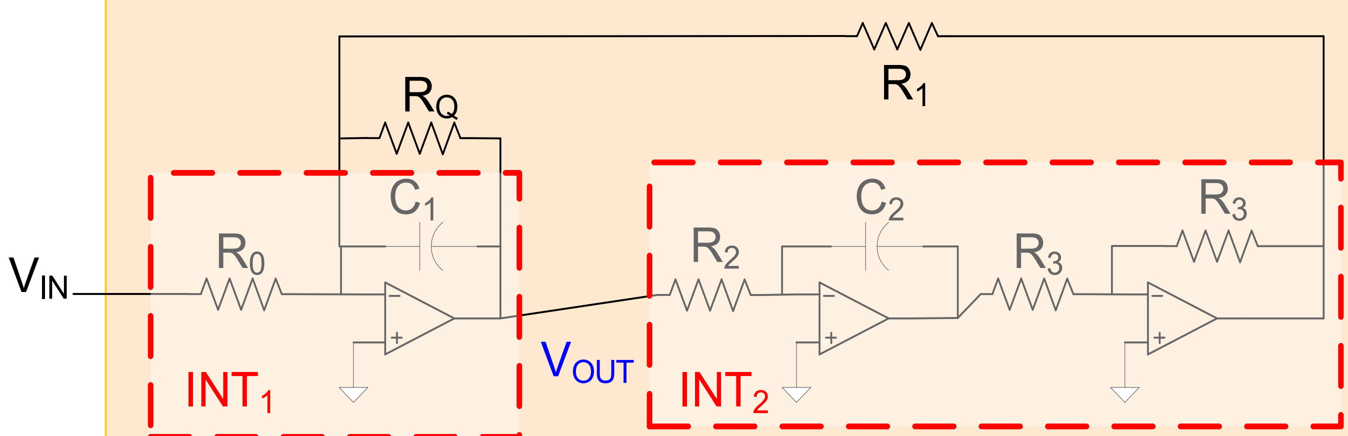
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R} \qquad BW = \left[\frac{R}{R_Q}\right]\frac{1}{RC}$$

Simple design process (sequential but not independent control of ω_0 and Q with R's, requires more tuning more than one R if C's fixed)

Modest component spread even for large Q

Example 5 (special case of previous filter):

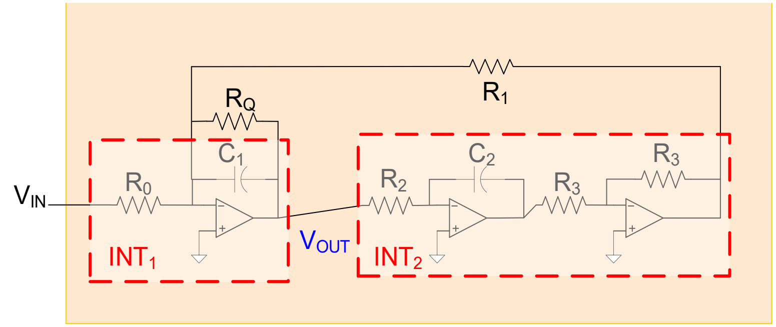


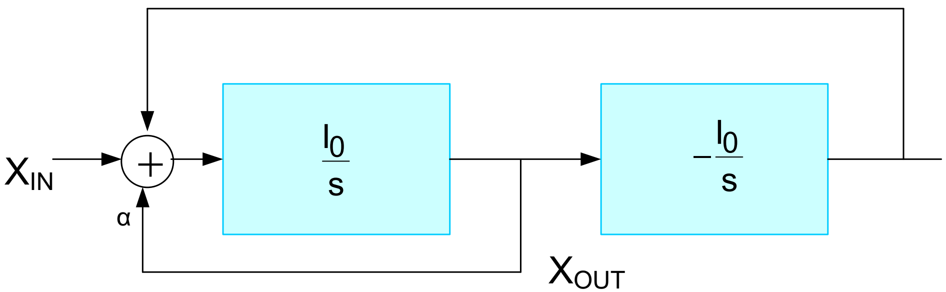


Two Integrator Loop Representation

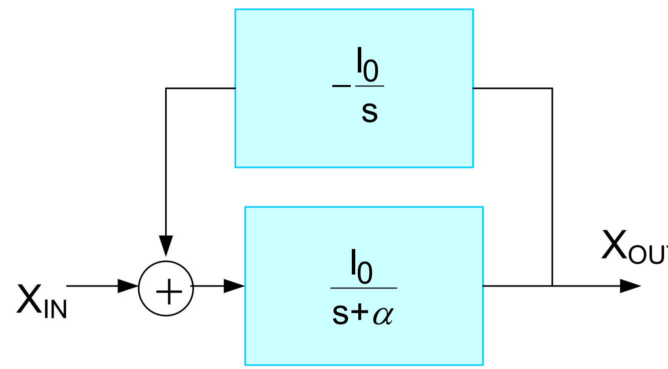
Example 5 (special case of previous filter):

Two Integrator Loop Representation

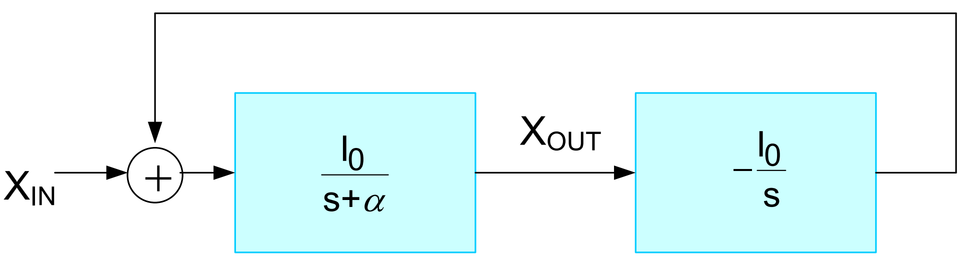




Inverting and Noninverting Integrator Loop

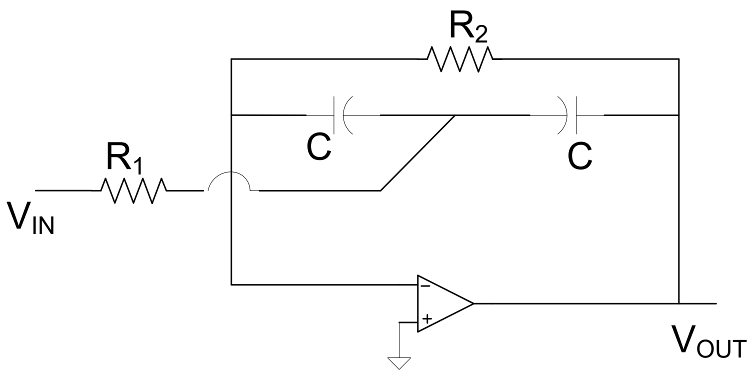


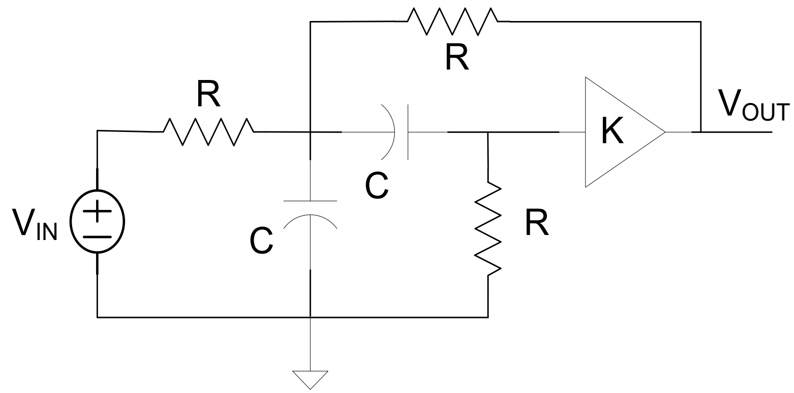
Integrator and Lossy Integrator Loop

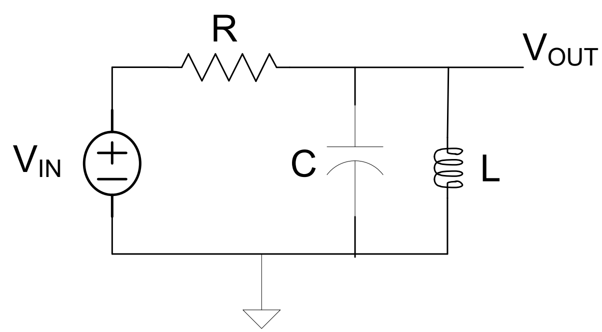


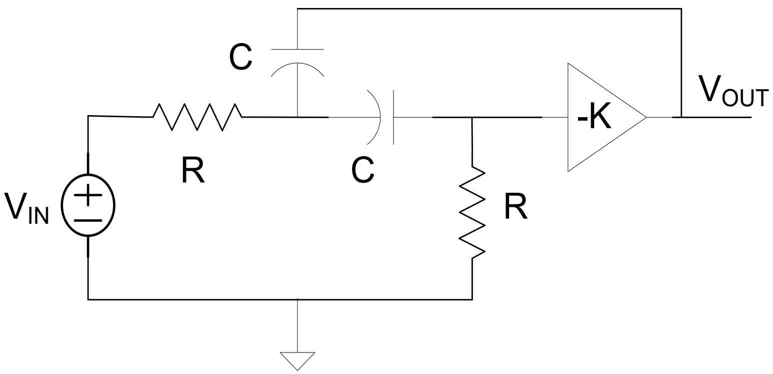
Integrator and Lossy Integrator Loop

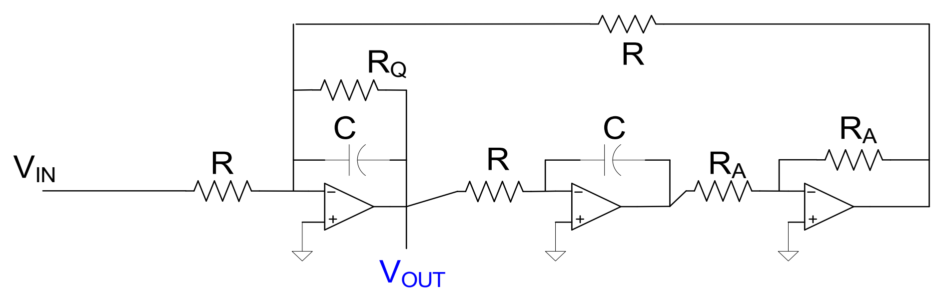
How does the performance of these bandpass filters compare?





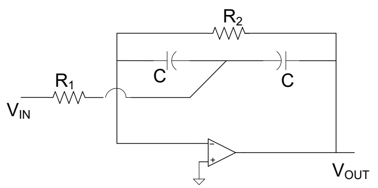


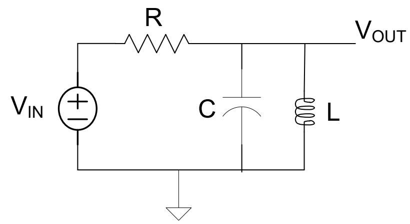


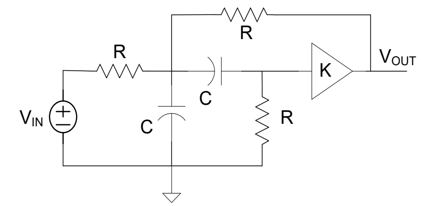


Ideally, all give same performance (within a gain factor)

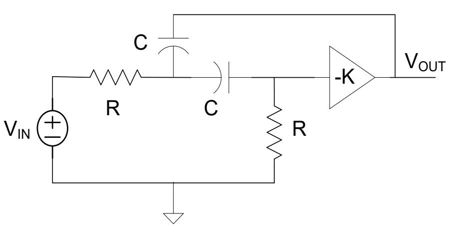
How does the performance of these bandpass filters compare?

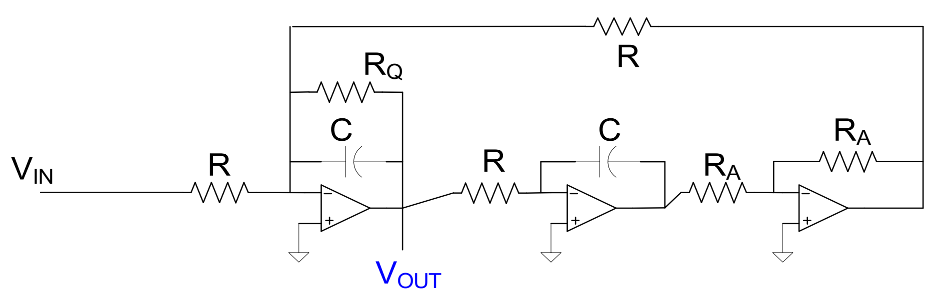






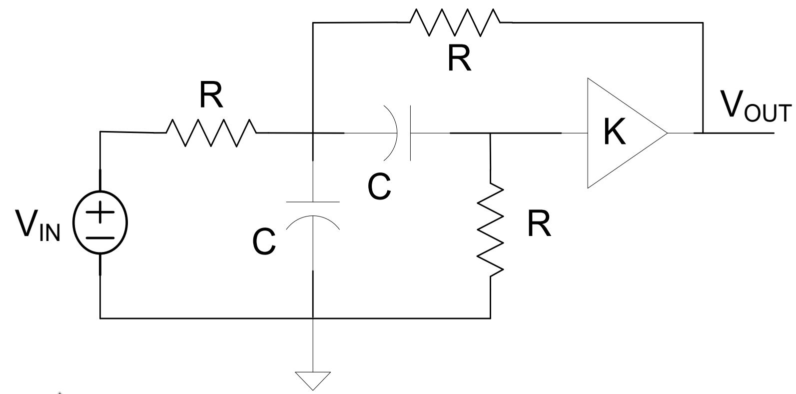
- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps



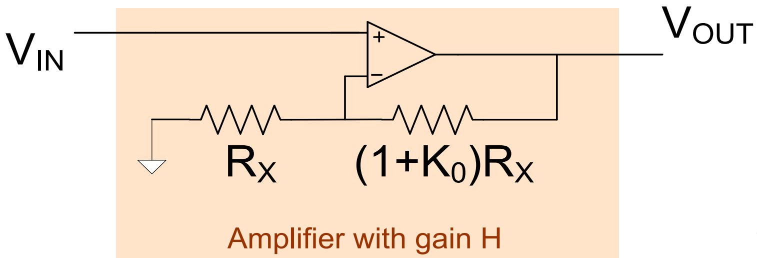


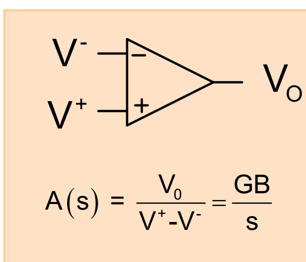
Consider effects of Op Amp on +KRC Bandpass with Equal R, Equal C

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K}$$



Assume K realized with standard Op Amp Circuit





$$K(s) = \frac{K_0}{1+\frac{K_0}{GB}s}$$

- Significant shift in peak frequency
- BW does not change very much
- Some drop in gain at peak frequency

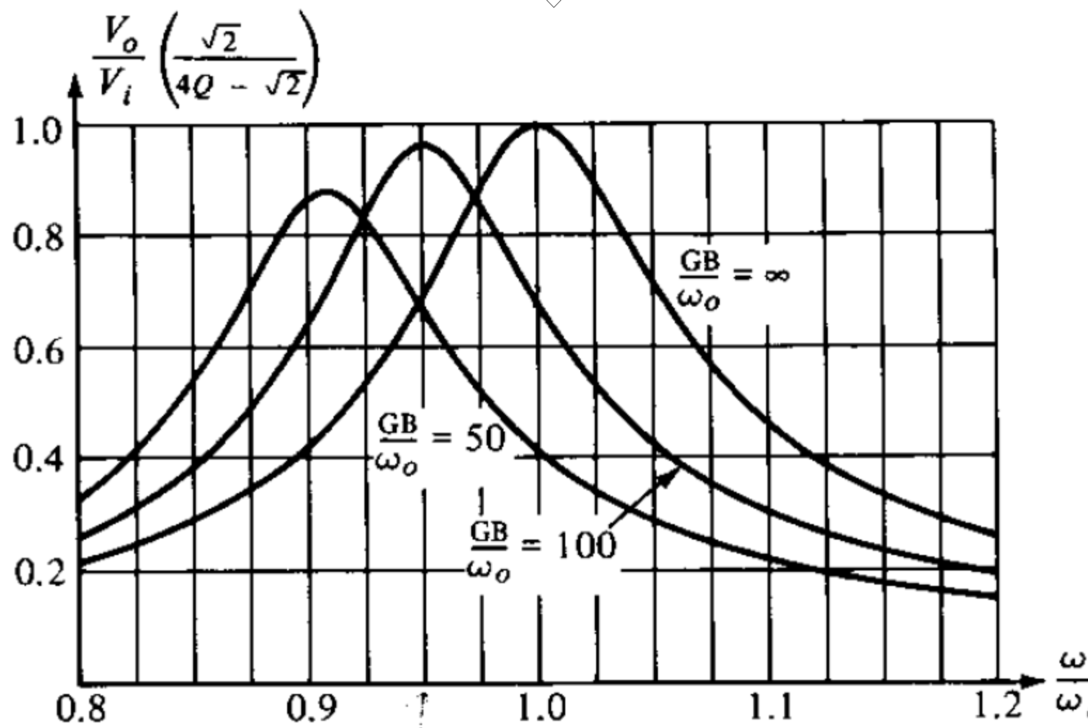


Fig. 11-4 Effect of GB on the magnitude curve for $Q = 10$

Practically, GB/ω_0 must be must larger than 100 for this filter

Stay Safe and Stay Healthy !

End of Lecture 17